Outline

Intrinsic Flame Instabilities

- Lecture 5/6 (March 14/16)
  - Saffman-Taylor instability
  - Additional Issues
    - Turbulent burning velocity
    - Diffusion flame instability
    - Edge flame instability
    - Diffusion flames in micro-combustors
  - Flame stretch: mathematical definition
  - Markstein number and relevance to turbulent combustion
Additional instability mode for flames propagating in a narrow channel (Hele-Shaw cell)

- Potential Application
  - Micro-combustor (in premixed combustion mode)
  - Combustion in engine crevice

Saffman-Taylor Instability

Saffman & Taylor (1958)

- Experimental Observations & Analytical Study
  - Two immiscible fluids in a Hele-Shaw cell.
  - Instability when the driving fluid is less viscous.
Joulin & Sivashinsky (1994)
• Linear stability analysis using
  - Pseudo-2D Euler-Darcy equations
    (z-direction parameterized – to be shown later)
Diffusive-thermal instability not considered

\[ \frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial z}(\rho v) = 0 \]

\[ \rho \left( \frac{\partial u}{\partial t} + u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} \right) = -\frac{\partial p}{\partial x} + fu \pm \rho g \quad (+: \text{upward}, -: \text{downward}) \]

\[ \rho \left( \frac{\partial v}{\partial t} + u \frac{\partial v}{\partial x} + v \frac{\partial v}{\partial y} \right) = -\frac{\partial p}{\partial y} + f v \quad f \sim \frac{\mu}{d^2} : \text{momentum loss} \]

\[ \rho c_p \left( \frac{\partial T}{\partial t} + U \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = -h(T - T_e) \quad h \sim \frac{\lambda}{d^2} : \text{heat loss} \]

Dispersion Relation
For small heat release: \( \gamma = \frac{\rho_u - \rho_b}{\rho_u} \ll 1, \quad f_{av} = \frac{1}{2}(f_b + f_u) \)

\[ \omega \sim \frac{\gamma U}{2} \left( \frac{\rho_u}{\rho} \frac{\rho_u U}{U} \left| k + \frac{h}{c_p} \right| \right) \]

2. Momentum loss enhances instability \( (f_b - f_u > 0 \text{ for gases}) \)

1. Heat loss reduces D-L instability

3. Gravity amplifies (upward) or attenuates (downward) the effect
Fundamentals of Flame Stabilities

S-T Instability Mechanism

- Increase in viscous friction due to increased velocity and viscosity
  \( \Rightarrow \text{Inherently hydrodynamic} \)
- Friction force does not attenuate with formation of cusp.
  \( \Rightarrow \text{S-T instability continues to grow after D-L reaches equilibrium.} \)

Fundamentals of Flame Stabilities

Computational Configuration

- Two-dimensional calculation
- One-step Arrhenius reaction
- Temperature-dependent viscosity

Galilean transformation
Assume Poiseuille flow (parabolic) in z-direction

\[ \overline{u}(x, y) = \int_0^1 u(x, y, z) \, dz \]

\[ u(x, y, z) = -\frac{6\{\overline{u}(x, y) - U_z\}}{h^2} z^2 + \frac{6\{\overline{u}(x, y) - U_z\}}{h} z + U_z \]

\[ \tau_{zz} = \mu \frac{\partial u}{\partial z} = -\mu \left[ \frac{12\{\overline{u}(x, y) - U_z\}}{h^2} z - \frac{6\{\overline{u}(x, y) - U_z\}}{h} \right] \]

\[ \frac{\partial \tau_{zz}}{\partial z} = \mu \frac{\partial^2 u}{\partial z^2} = -\frac{12\mu\{\overline{u}(x, y) - U_z\}}{h^2} \]

Effects of heat loss

\[ T(x, y, z) = -\frac{6\{\overline{T}(x, y) - T_z\}}{h^2} z^2 + \frac{6\{\overline{T}(x, y) - T_z\}}{h} z + T_u \]

\[ T_u = T_0 + (1 - H)(T_h - T_0) \]

Solve for z-averaged variables \((\overline{\rho}, \overline{u}, \overline{v}, \overline{T}, \overline{Y})\)

**Conservation Equations**

\[ \frac{\partial \overline{\rho}}{\partial t} + \frac{\partial (\overline{\rho} \overline{u})}{\partial x} + \frac{\partial (\overline{\rho} \overline{v})}{\partial y} = 0 \]

**mass**

\[ \frac{\partial (\overline{u})}{\partial t} + \frac{\partial (\overline{u} \overline{u})}{\partial x} + \frac{\partial (\overline{v} \overline{u})}{\partial y} = \frac{1}{Re}\left[ \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y} \right] \]

**x-momentum**

\[ \frac{\partial (\overline{p})}{\partial t} + \frac{\partial (\overline{p} \overline{u})}{\partial x} + \frac{\partial (\overline{p} \overline{v})}{\partial y} = \frac{1}{Re}\left[ \frac{\partial \tau_{px}}{\partial x} + \frac{\partial \tau_{vx}}{\partial y} \right] \]

**y-momentum**

\[ \frac{\partial (\overline{E})}{\partial t} + \frac{\partial (\overline{E} \overline{u})}{\partial x} + \frac{\partial (\overline{E} \overline{v})}{\partial y} = \frac{1}{RePr}\left[ \frac{\partial \tau_{ex}}{\partial x} + \frac{\partial \tau_{ey}}{\partial y} \right] + \frac{\partial \overline{Q}}{\partial t} \]

**energy**

\[ \frac{\partial (\overline{S})}{\partial t} + \frac{\partial (\overline{S} \overline{u})}{\partial x} + \frac{\partial (\overline{S} \overline{v})}{\partial y} = \frac{1}{ReSc}\left[ \frac{\partial \tau_{sx}}{\partial x} + \frac{\partial \tau_{sv}}{\partial y} \right] + \frac{\partial \overline{Q_s}}{\partial t} \]

**species**

where \( \overline{Q} = \Delta \overline{P}_x \exp\left( -\frac{\beta(1 - \theta)}{1 - \alpha(1 - \theta)} \right) \)

\[ \beta = \frac{E_P}{RT_i^2} (\overline{T} - \overline{T}_i) \]

\[ \alpha = \frac{\overline{T} - \overline{T}_i}{\overline{T}_i} \]
**Fundamentals of Flame Stabilities**

**Dispersion Relation**

- **Key parameter:** \( \text{Pe} = \frac{S_L h}{\alpha} \)

### Diffusive-Thermal

- \( Le = 0.7 \)
- \( Le = 1.0 \)
- \( Le = 1.3 \)

### Saffman-Taylor

- No S-T effect
- \( h^+ = 2.0 \)
- \( h^+ = 1.8 \)
- \( h^+ = 1.5 \)

Wave number selection is similar to D-L

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**Fundamentals of Flame Stabilities**

**Effect of Peclet Number**

- **Normalized growth rate (at maximum \( k \))**

For S-T effect to be important:

- \( \text{Pe} < 50 \)
- \( h / \delta_{th} < 10 \)
- No clear dependence of $S_T/S_L$ on Pe
- Experimentally limited to high Pe

Previous Experiments

- D-L & D-T only
- Heat Loss only
- Heat & Mom Loss
- Mom Loss only
### Fundamentals of Flame Stabilities

#### Summary – Cell Formation Behavior

<table>
<thead>
<tr>
<th></th>
<th>Le OTHER EFFECTS</th>
<th>Le 0.7</th>
<th>Le 1.0</th>
</tr>
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<tr>
<td>Heat Loss</td>
<td>3~4 cells</td>
<td>2 cells</td>
<td></td>
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<tr>
<td>Normal</td>
<td>2 cells</td>
<td>1 cell</td>
<td></td>
</tr>
<tr>
<td>Momentum Loss</td>
<td>1 cell</td>
<td>1 cell</td>
<td></td>
</tr>
</tbody>
</table>

$\lambda^{**} = 20$, $F/\lambda = 10^{-2}$, $(h^{**}=7$, $H=0.5)$

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### Some Advanced Subjects on Flame Instabilities
Fundamentals of Flame Stabilities
Additional Issues - Turbulent Combustion

Turbulent Burning Velocity
Existing theories and experiments don’t agree among one another.

\[
\frac{S_T}{S_L} = F\left(\frac{u'}{S_L}\right)
\]

The cause of the “bending” effects remains an open issue.
(Ronney, 1994)

Fundamentals of Flame Stabilities
Effects of Thermal Expansion

Cambray & Joulin (1992)
- \(\frac{S_T}{S_L}\) increases as \(\gamma = \frac{\rho_u - \rho_h}{\rho_u}\) increases.
- Thermal expansion-induced wrinkling (D-L instability) is more pronounced for low turbulence intensity.
⇒ Possible explanation for bending effects
Nonpremixed Flame Instabilities
- Some experimental observations
- Cell structures typically observed in the direction of no flow straining
- Asymptotic analysis showed similar types of diffusion flame instabilities (Kim, 1997)
  • Cellular instability for $\text{Le} < 1$
  • Pulsating instability near extinction condition for $\text{Le} > 1$.

Edge Flame Instabilities (Thatcher & Dold, 2001)
- $S_e > 1$ for $L < L_c$
- Edges continue to propagate even when the trailing diffusion flame no longer exists (in a temporally oscillating mode).
Nonpremixed Combustion in a Microchannel
(Miese et al., 2005)
- Cell structures observed in a narrow channel subjected to heat loss
- Number of cells depends on mixture and flow conditions.

Flame Stretch:
Effects of Aerodynamics on Flame Propagation
In many flame instability studies, it was essential to consider that flame speed depends on flow conditions.

- Markstein
  - Flame speed depends on curvature
- Diffusive-thermal instability:
  - Flame speed changes due to imbalance between mass and heat transport (Lewis number)

⇒ Formal theoretical description of the net effects of flow field on flame speed is needed.

**Fundamentals of Flame Stabilities**

**Flame Stretch - Definition**

Karlovitz (1953): \[ \kappa = \frac{dU}{dy} \]

Williams (1975):
\[ \kappa = \frac{1}{A} \frac{dA}{dt}, \quad Ka = \frac{\delta_t \kappa}{S_L} \quad \text{(Karlovitz Number)} \]

**Kinematic consideration**

- \( n \): unit normal vector
- \( V_f \): surface velocity (lab frame)
- \( v \): flow velocity (lab frame)
- \( e_p, e_q \): tangent vectors on surface

\[ S_L = \left[ V_f \cdot n - v \cdot n \right]_{G=0} \]
### Flame Stretch - Definition

**A. Matalon (1983)**

$$\kappa = - \left\{ \nabla \times (v \times n) \right\}_{G=0} \cdot n - \left( V_f \cdot n \right) \left( \nabla \cdot n \right)_{G=0}$$

**B. Chung & Law (1984)**

$$\kappa = \nabla \cdot v_{s,t} + \left( V_f \cdot n \right) \left( \nabla \cdot n \right)$$

where $v_{s,t} = V_t$; $V_f = V_j + \left( V_f \cdot n \right) n$

**C. Candel & Poinsot (1990)**

$$\kappa = -n n \nabla v + \nabla \cdot v + S_t \left( \nabla \cdot v \right)$$

$$= \left( e_x e_x + e_y e_y \right) : \nabla v + S_t \left( \nabla \cdot n \right) = a + S_t \left( \nabla \cdot n \right)$$

Note that in A & B, curvature effects may be included in the first term.

### Flame Stretch – Physical Implication

**Chung & Law (1984)**

$$\kappa = \nabla \cdot v_{s,t} + \left( V_f \cdot n \right) \left( \nabla \cdot n \right)$$

(1) Flow nonuniformity along the flame surface

$$v_{s,t} = n \times (v \times n) \neq 0 \quad \text{only if flow is oblique to the surface}$$

(2) $V_f$: Flow velocity in the inertial frame

$$V_f \cdot n = 0 \quad \text{for stationary flames}$$

(3) $\nabla \cdot n \neq 0 \quad \text{only for curved flames.}$
Examples

\[ \kappa = \nabla \cdot \mathbf{v}_{x,t} + \left( \mathbf{V}_f \cdot \mathbf{n} \right) \left( \nabla \cdot \mathbf{n} \right) \]

(a) Spherical expanding flame
\[ \mathbf{v}_{x,t} = 0, \text{ but } \nabla \cdot \mathbf{n} \neq 0, \mathbf{V}_f \cdot \mathbf{n} \neq 0 \implies \kappa \neq 0 \]

(b) Steady spherical flame
\[ \mathbf{V}_f \cdot \mathbf{n} = 0, \nabla \cdot \mathbf{v}_{x,t} = 0 \implies \kappa = 0 \]

(c) Steady curved flame in uniform flow
\[ \mathbf{V}_f \cdot \mathbf{n} = 0, \text{ but } \nabla \cdot \mathbf{v}_{x,t} \neq 0 \implies \kappa \neq 0 \]

Steady Stagnation Flow (Hiemenz)

\[ \kappa = \nabla \cdot \mathbf{v}_{x,t} + \left( \mathbf{V}_f \cdot \mathbf{n} \right) \left( \nabla \cdot \mathbf{n} \right) \]

\[ \mathbf{v} = \left( \frac{a}{k+1} x, -ay, 0 \right)_{x,t} = 0, \quad \begin{cases} a : \text{strain rate (s}^{-1}\text{)} \\ k = \begin{cases} 0 : 2-\text{D slab} \\ 1 : \text{axisymmetric} \end{cases} \end{cases} \]

\[ \mathbf{n} = \mathbf{j}, \mathbf{e}_t = \mathbf{i} \]

\[ \Rightarrow \mathbf{V}_f \cdot \mathbf{n} = 0, \nabla \cdot \mathbf{v} = \left[ \frac{\partial}{\partial x} \left( ax \right) = a \quad (2D) \right] \]

\[ \frac{1}{r} \frac{\partial}{\partial r} \left[ \frac{1}{2} ar^2 \right] = a \quad \text{(axisymmetric)} \]

\[ \kappa = a \]
Fundamentals of Flame Stabilities

Flame Stretch – Further Examples

Spherical Flames

$$\kappa = \nabla \cdot \mathbf{v}_{s,j} + \left( \mathbf{V}_f \cdot \mathbf{n} \right) \left( \nabla \cdot \mathbf{n} \right)$$

$$\mathbf{v}_{s,j} = 0, \quad \mathbf{V}_f \cdot \mathbf{n} = \frac{dR_f}{dt}$$

$$\nabla \cdot \mathbf{n} = e_p \cdot \frac{\partial \mathbf{n}}{\partial p} + e_q \cdot \frac{\partial \mathbf{n}}{\partial q} = \frac{\partial \left( e_p \cdot \mathbf{n} \right)}{\partial p} + \frac{\partial \left( e_q \cdot \mathbf{n} \right)}{\partial q} - \mathbf{n} \cdot \left( \frac{\partial e_p}{\partial p} + \frac{\partial e_q}{\partial q} \right) = \frac{1}{R_1} + \frac{1}{R_2}$$

$$\kappa = \frac{2}{R_f} \frac{dR_f}{dt}$$

> 0 for outward prop.

< 0 for inward prop.

Bunsen Flame

$$\kappa = \nabla \cdot \mathbf{v}_{s,j} + \left( \mathbf{V}_f \cdot \mathbf{n} \right) \left( \nabla \cdot \mathbf{n} \right)$$

$$\mathbf{V}_f \cdot \mathbf{n} = 0, \quad \mathbf{v} = (0, 0, w), \quad \mathbf{n} = (\cos \alpha, 0, -\sin \alpha)$$

$$\kappa = -\left[ \frac{\sin \alpha}{r} \frac{\partial}{\partial r} \left( rw \cos \alpha \right) + \cos \alpha \frac{\partial}{\partial z} \left( w \cos \alpha \right) \right]$$

If \( \alpha = \text{constant (except at the tip)} \)

$$\kappa = -\frac{w \sin 2\alpha}{2R_f} \quad (< 0)$$
**Fundamentals of Flame Stabilities**

**Effects of Flame Stretch - Phenomenology**

**Key Parameters:**

\[
\alpha = \frac{\hat{\lambda}}{\rho c_p}, \ D_i (\text{deficient}), \ D_j (\text{excess})
\]

\[
\text{Le}_i = \frac{\alpha}{D_i} (\text{Lewis number}); \ D_i / D_j (\text{Preferential diffusion})
\]

**Example: Bunsen flame tip**

- \(\text{Le}_i = 1\): no effect
- \(\text{Le}_i > 1\): stronger tip (lean propane/air)
- \(\text{Le}_i < 1\): tip opening (lean hydrogen/air)

- \(D_i / D_j = 1\): no effect
- \(D_i / D_j < 1\): stronger tip (rich hydrogen/air)
- \(D_i / D_j > 1\): tip opening (rich propane)

**Fundamentals of Flame Stabilities**

**Effects of Flame Stretch for Bunsen Flames**

**Experimental Observation (Law & Sung, 2000)**

- (a) rich \(\text{C}_3\text{H}_8/\text{air}\)
- (b) lean \(\text{C}_3\text{H}_8/\text{air}\)
- (c) rich \(\text{CH}_4/\text{air}\)
- (d) lean \(\text{CH}_4/\text{air}\)
**Fundamentals of Flame Stabilities**

**Effects of Flame Stretch - Counterflow**

**Example: Counterflow flames ($\kappa > 0$)**

- $L_e = 1$: no effect
- $L_e > 1$: weaker flame
  - (lean propane/air)
- $L_e < 1$: stronger flame
  - (lean hydrogen/air)

- $D_i / D_j = 1$: no effect
- $D_i / D_j < 1$: weaker flame
  - (rich hydrogen/air)
- $D_i / D_j > 1$: stronger flame
  - (rich propane)

**Fundamentals of Flame Stabilities**

**Effects of Flame Stretch in Counterflow**

**Numerical Results: Flame Temperature vs. Stretch**

- $T_s$ (K)

- $\phi=0.52$
- $\phi=1.62$
- $\phi=0.58$
- $\phi=2.25$
Fundamentals of Flame Stabilities

Effects of Flame Stretch on Flame Speed

Markstein (1950)

\[ \frac{S_L}{S_{L,K=0}} = 1 - \mu \nabla f \]  
(heuristic curvature effect)

Asymptotic Analysis for Low Stretch Flames

\[ \frac{S_L}{S_{L,K=0}} = 1 - L_K + \cdots = 1 - \text{MaKa} \]

\[ \text{Ma} = \frac{L}{\delta} \]  
Markstein number; \( \text{Ka} = \frac{\delta K}{S_L} \)  
Karlovitz number

Clavin & Williams (1982), Clavin & Garcia (1983)

\[ \text{Ma} = \frac{1}{\gamma} J + \beta (L e - 1) \left( \frac{1 - \gamma}{2} \right) D \]

\[ D = \int_{T_b}^{T_u} \frac{\lambda}{\beta(T)} \ln \left( \frac{T_b - T_u}{T - T_b} \right) dT \]

Empirical Corrections for Larger Ka

Faeth et al.

\[ \frac{S_{L,K=0}}{S_L} = 1 + \text{MaKa} \]

Poinsot et al.

\[ \frac{S_L}{S_{L,K=0}} = 1 - \left( \frac{L}{S_{L,K=0}} \right) C \nabla \cdot \mathbf{v} \]  
\( C = \frac{\nabla \cdot \mathbf{v}}{1 - C \left( \frac{L}{R} \right)} \)  
\( R: \text{radius of curvature} \)

Further ambiguities
- Definition of flame speed
- Definition of flame thickness
**Fundamentals of Flame Stabilities**

**Flame Speed Correlation**

**Extension to Turbulent Flames (Chen & Im, 2000)**

- Two branches in correlation curves
- Effects of unsteadiness at high turbulence

**Unsteady Effects (Im & Chen, 2000)**

- Flame response to harmonic oscillation in strain rate

\[ a = a_0 (1 + A \sin \omega t) \]

\[ M = \frac{S_{L,\text{max}} - S_{L,\text{min}}}{K a_{\text{max}} - K a_{\text{min}}} = F(\omega) \]

Markstein transfer function
Fundamentals of Flame Stabilities

References (1)


References (2)